Seat No.	

M.A./M.Sc. (Semester – II) Examination, 2012
MATHEMATICS
MT-605 : Partial Differential Equations
(2008 Pattern)

Time : 3 Hours   Max. Marks : 80

N.B. : *i) Attempt* ***any five*** *questions.*
ii) Figures to the ***right*** *indicate* ***full*** *marks.*

1. a) Eliminate the arbitrary function f from the equation $x + y + z = f(x^2 + y^2 + z^2)$ **5**

   b) Find the general solution of :
   $y^2p - xyq = x(z - 2y)$. **5**

   c) Solve $\frac{\partial^2 z}{\partial x \partial y} = x^2 y$. **3**

   d) State the condition for the equations $f(x, y, z, p, q) = 0$ and $g(x, y, z, p, q) = 0$ to be compatible on a domain D. **3**

2. a) Solve the nonlinear partial differential equation $zpq - p - q = 0$. **4**

   b) Explain the method of solving the first order partial differential equations.
   i) $f(z, p, q) = 0$
   ii) $g(x, p) = h(y, q)$. **6**

   c) Find a one parameter family of common solutions of the equations $xp = yq$ and $z(xp + yq) = 2xy$. **6**

3. a) If an element $(x_0, y_0, z_0, p_0, q_0)$ is common to both an integral surface $z = z(x, y)$ and a characteristic strip, then show that the corresponding characteristic curve lies completely on the surface. **8**

   b) Find the solution of $z = \frac{1}{2}(p^2 + q^2) + (p - x)(q - y)$ which passes through the x-axis. **8**

P.T.O.

4. a) Find the complete integral of $p^2 - y^2 q = y^2 - x^2$ by Charpits method. **7**

b) Reduce $\frac{\partial^2 u}{\partial x^2} = (1 + y)^2 \frac{\partial^2 u}{\partial y^2}$ to canonical form . **6**

c) Prove that the solution of Neumann problem is unique up to the addition of a constant. **3**

5. a) Using D'Alemberts solution of infinite string find the solution of

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}, \ 0 < x < \infty, \ t > 0$$

$y(x, 0) = u(x)$, $y_t(x, 0) = v(x)$, $x \geq 0$

$y(0, t) = 0$, $t \geq 0$. **8**

b) Solve the equation $U_x^2 + U_y^2 + U_z = 1$ by Jacobi's method. **8**

6. a) Prove that the solution of following problem exist then it is unique :

$u_{tt} - c^2 u_{xx} = F(x, t)$, $0 < x < l$, $t > 0$

$u(x, 0) = f(x)$ $\quad 0 \leq x \leq l$

$u_t(x, 0) = g(x)$

$u(0, t) = u(l, t) = 0$, $t \geq 0$. **6**

b) Suppose that u(x, y) is harmonic in a bounded domain D and continuous in $\overline{D}$ = DUB then prove that u attains its maximum on the boundary B of D. **6**

c) Classify the equation $u_{xx} - 2x^2 u_{xz} + u_{yy} + u_{zz} = 0$ into hyperbolic, parabolic or elliptic type. **4**

7. a) State and prove Harnack's theorem. **6**

b) Find the solution of Dirichlet problem for the upper half plane which is defined as $u_{xx} + u_{yy} = 0$; $-\infty < x < \infty$, $y > 0$

$u(x, 0) = f(x)$ $-\infty < x < \infty$ with the condition that u is bounded as $y \to \infty$, u and u_x vanish as $|x| \to \infty$. **6**

c) Solve the Quasi-Linear equation $zz_x + z_y = 1$ containing the initial data curve $x_0 = s$, $y_0 = s$, $z_0 = \frac{1}{2} s$ for $0 \leq s \leq 1$. **4**

8. a) Using Duhamel's principle find the solution of non homogeneous equation

$u_{tt} - c^2 u_{xx} = f(x, t)$; $-\infty < x < \infty$, $t > 0$

$u(x, 0) = u_t(x, 0) = 0$; $-\infty < x < \infty$. **8**

b) Using the variable separable method solve $u_t = ku_{xx}$; $0 < x < a$, $t > 0$ which satisfies condition $u(0, t) = u(a, t) = 0$; $t > 0$ and $u(x, 0) = x(a - x)$; $0 \leq x \leq a$. **8**

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Seat No.	

M.A./M.Sc. (Semester – I) Examination, 2012
MATHEMATICS
(2008 Pattern)
MT-502 Advanced Calculus

Time : 3 Hours Max. Marks : 80

N.B. : *1) Attempt* ***any five*** *questions.*
2) Figures at ***right*** *indicate* ***full*** *marks.*

1. a) Show that composition of continuous functions is continuous. **5**

b) Compute all first order partial derivatives for function $f(x, y) = x^4 + y^4 – 4x^2y^2$ what can you say about their mixed partial derivatives ? **5**

c) With all usual notations prove that $T_a(\bar{y}) = f'(\bar{a}; \bar{y})$. **6**

2. a) Find the directional derivative of scalar field $f(x, y) = x^2 – 3xy$ along the parabola $y = x^2 – x + 2$ at the point (1, 2). **5**

b) Comment : If the vector field $\bar{f}$ is differentiable at $\bar{a}$, then $\bar{f}$ is continuous at $\bar{a}$. **5**

c) State and prove matrix form of chain rule. **6**

3. a) Let f be two dimensional vector field given by $f(x,y) = \sqrt{y}\ i + (x^3 + y)\ j\ \forall\ (x,y)\ y \geq 0$ calculate line integral of f from (0, 0) to (2, 2) along straight line joining these two points. **5**

b) Prove that the change in kinetic energy in any time interval is equal to work done by f during this time interval. **5**

c) State and prove second fundamental theorem of calculus. **6**

4. a) State and prove linearity property of double integral. **5**

b) Evaluate : $\iint_Q (x \sin y - y\ e^x)\ dx\ dy$ where $Q = [-1, 1] \times [0, \pi/2]$ Figure is expected. **5**

c) Show that graph of continuous real valued function on closed interval has content zero. **6**

P.T.O.

5. a) State and prove Green's theorem for plane regions bounded by Peicewise Smooth Jordon curves. **8**

   b) Evaluate the integral $\iint_S e^{(y-x)|(y+x)}\,dx\,dy$ where S is triangle bounded by line x + y = 2 and two coordinate axes. **5**

   c) State first fundamental theorem of calculus. **3**

6. a) Show that Jacobian of transformation by spherical coordinates with all usual notations is $-\rho^2 \sin\phi$. **5**

   b) Write the parametric representation of a surface of sphere. **5**

   c) Define fundamental vector product. Explain its geometrical interpretation. **6**

7. a) Find the area of hemisphere of radius 1 using surface integrals. **5**

   b) Define surface integral and illustrate with an example. **3**

   c) State and prove Stroke's theorem. **8**

8. a) Find divergence and curl of a gradient of scalar field. **6**

   b) State and prove divergence theorem. **10**

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Seat No.	

M.A./M.Sc. (Semester – I) Examination, 2012
MATHEMATICS
MT 505 : Ordinary Differential Equations
(2008 Pattern)

Time : 3 Hours Max. Marks : 80

N.B. *: i) Attempt* ***any five*** *questions.*
ii) Figures to the ***right*** *indicate* ***full*** *marks.*

1. a) If $y_1(x)$ and $y_2(x)$ are any two solutions of equation $y'' + P(x)y' + Q(x)y = 0$ on [a, b] then prove that their Wronskian W = W (y_1, y_2) is either identically zero or never zero on [a, b]. **6**

b) If $y_1 = x$ is one solution of $x^2y'' + xy' - y = 0$ then find other solution. **6**

c) Verify that $y_1 = 1$ and $y_2 = \log x$ are linearly independent solutions of a equation $y'' + (y')^2 = 0$ on any interval to the right of the origin. **4**

2. a) Discuss the method of variation of parameters to find the solution of second order differential equation with constant coefficients. **8**

b) Find the general solution of $y'' - y' - 2y = 4x^2$ by using method of undetermined coefficients. **6**

c) Reduced the equation $x''(t) + 4t\, x'(t) + t^2\, x = 0$ into an equivalent system of first order equation. **2**

3. a) State and prove sturm comparison theorem. **8**

b) Find the general solution of a equation $(1 - x^2)y'' - 2xy' + P(P+1)y = 0$ about X = 0 by power series method. **8**

4. a) Let u(x) be any nontrivial solution of $u'' + q(x)u = 0$ where q(x) > 0 for all x > 0. If $\int_1^\infty q(x)\,dx = \infty$ then prove that u(x) has infinitely many zeros on the positive x-axis. **8**

b) Find the solution of $y'' - 5y' + 6y = 0$ with initial condition y(1) = e^2 and $y'(1) = 3e^2$. **4**

c) Locate and classify the singular points on the x-axis of a equation $x^2(x^2-1)^2\ y'' - x(1-x)\,y' + 2y = 0$. **4**

5. a) Solve the system

$$\frac{dx}{dt} = x + y$$

$$\frac{dy}{dt} = 4x - 2y$$ **8**

b) Find the critical points of

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} = x^3 + x^2 - 2x.$$ **5**

c) Determine the nature of a point $x = \infty$ for the equation $x^2y'' + 4xy' + 2y = 0$. **3**

6. a) If m_1 and m_2 are roots of the auxiliary equation of the system

$$\frac{dx}{dt} = a_1x + b_1y$$

$$\frac{dy}{dt} = a_2x + b_2y$$

Which are real, distinct and of the same sign then prove that the critical point (0, 0) is a nod ? **8**

b) Find all solutions of the nonautonomous system $\frac{dx}{dt} = x;\ \frac{dy}{dt} = x + e^t$ and sketch some of the curves defined by these solutions. **8**

7. a) Determine whether the following functions is positive definite, negative definite or neither with justification.

   i) $x^2 - xy - y^2$

   ii) $2x^2 - 3xy + 3y^2$

   iii) $-x^2 - 4xy - 5y^2$ **8**

   b) Solve the following initial value problem by Picards method

   $y' = x + y\ ;\ y(0) = 1$ **6**

   c) State Picard's existence and uniqueness theorem. **2**

8. a) If f(x, y) be a continuous function that satisfies a Lipschitz condition.

   $|f(x, y_1) - f(x, y_2)| \le k|y_1 - y_2|$ on a strip defined by $a \le x \le b$ and $-\infty < y < \infty$. If (x_0, y_0) is any point of the strip, then prove that the initial value problem $y' = f(x, y), y(x_0) = y_0$ has one and only one solution y = f(x) on the interval $a \le x \le b$. **10**

   b) Solve the following initial value problem

   $$\frac{dy}{dx} = z \qquad y(0) = 1$$

   $$\frac{dz}{dx} = -y \qquad z(0) = 0$$ **6**

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Seat No.	

M.A./M.Sc. (Semester – II) Examination, 2012
MATHEMATICS
MT 603 : Groups and Rings
(2008 Pattern)

Time : 3 Hours Max. Marks : 80

N.B. *: 1) Attempt* ***any five*** *questions.*
2) Figures to ***right*** *indicate* ***full*** *marks.*
3) ***All*** *questions carry* ***equal*** *marks.*

1. a) If G is a finite cyclic group generated by 'a' and $|G| = n$ then prove that for each positive divisor K of n, G has exactly one subgroup of order K. **6**

   b) Prove that a group of order 4 is abelian. **5**

   c) If the group G has exactly two non-trivial proper subgroups then prove that G is cyclic and $|G| = pq$ where p and q are distinct primes or G in cyclic and $|G| = p^3$ where p is prime. **5**

2. a) If the pair of cycles $\alpha = (a_1, a_2, ..., a_m)$ and $\beta = (b_1, b_2, ..., b_n)$ have no entries in common then prove that $\alpha\beta = \beta\alpha$. **6**

   b) If $\beta \in S_7$ and if $\beta^4 = (2\ 1\ 4\ 3\ 5\ 6\ 7)$; then find β. **5**

   c) Prove that the cyclic group Z_n has even number of generators. If n > 2. **5**

3. a) Prove that every group is isomorphic to a group of permutation. **8**

   b) State the converse of Lagrange's theorem for finite group.
   Is the converse of Lagrange's theorem true ? Justify. **8**

4. a) Let G and H be finite cyclic groups prove that $G \oplus H$ is cyclic if and only if $|G|$ and $|H|$ are relatively prime. **6**

P.T.O.

b) Determine the number of elements of order 5 in $Z_{25} \oplus Z_5$. **5**

c) Prove or disprove :

i) $Z \oplus Z$ is cyclic

ii) $S_3 \oplus Z_2 \simeq A_4$. **5**

5. a) Prove that for any group G, $\frac{G}{Z(G)}$ is isomorphic to Inn (G). **6**

b) Let G be a non-abelian group of order p^3 (P is prime) and $Z(G) \neq e$ then prove that $|Z(G)| = p$. **5**

c) Find a group homomorphism ϕ from U(40) to U(40) with Kernel {1, 9, 17, 33} and $\phi(11) = 11$. **5**

6. a) If K is a subgroup of G and N is a normal subgroup of G then prove that $\frac{K}{K \cap N} \cong \frac{KN}{N}$. **6**

b) Determine all homomorphic images of D_4, octic group (upto an isomorphism). **5**

c) Prove that any Abelian group of order 45 has an element of order 15. Does every abelian group of order 45 have an element of order 9 ? **5**

7. a) If $|G| = p^2$, where p is prime then prove that G is abelian. **6**

b) Write the class equation of the group D_4 (oclic group) and hence find its all normal subgroups. **5**

c) Prove that $\frac{D_4}{Z(D_4)} \cong Z_2 \oplus Z_2$. **5**

8. a) If G is a finite group and p is a prime such that p^k divides $|G|$ then prove that G has at least one subgroup of order p^k. **6**

b) Show that there are only two abelian groups of order 99. Determine them. **5**

c) Determine the number elements of order 5 in a group of order 20. **5**

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Seat No.	

M.A./M.Sc. (Semester – II) Examination, 2012
Mathematics
MT-604 : COMPLEX ANALYSIS
(2008 Pattern)

Time : 3 Hours   Max. Marks : 80

N.B. *: 1) Answer **any five** questions.*
*2) Figures to the **right** indicate **full** marks.*

1. a) If Σ an $(z-a)^n$ is a given power series with radius of convergence R, then prove that $R = \lim \left|\frac{a_n}{a_{n+1}}\right|$ if this limit exists. **6**

b) Under stereographic projection for each of points z = 0, z = 3 + 2i, give the corresponding points of the unit sphere S in $\mathbb{R}^3$ **6**

c) Find the radius of convergence of $\sum_{n=1}^{\infty} \frac{(-1)^n}{n} z^{n(n+1)}$. **4**

2. a) Let f and g be analytic on G and Ω respectively and suppose f(G) C Ω. Prove that gof is analytic on G and $(gof)'(z) = g'(f(z))f'(z)\ \forall z \in G$. **8**

b) Let G be either the whole plane $\mathbb{C}$ or some open disk. If $u : G \to \mathbb{R}$ is a harmonic function then prove that u has a harmonic conjugate. **5**

c) Show that $f(z) = |z|^2 = x^2 + y^2$ has a derivative only at the origin. **3**

3. a) Define a Mobius transformation. If z_2, z_3, z_4 are distinct points and T is any Mobius transformation then prove that $(z_1, z_2, z_3, z_4) = Tz_1, Tz_2, Tz_3, Tz_4)$ for any point z_1. **6**

b) Let f be analytic in the disk B(ajR) and suppose that γ is a closed rectifiable curve in B(ajR). Prove that $\int_\gamma f = 0$. **5**

c) Let $\gamma(t) = e^{it}$ for $0 \le t \le 2\pi$. Then find $\int_\gamma z^n dz$ for every integer n. **5**

P.T.O.

4. a) Let G be a connected open set and let $f : G \to \mathbb{C}$ be an analytic function. Prove that the following are equivalent statement

   i) $f \equiv 0$

   ii) There is a point a in G such that $f^n(a) = 0$ for each $n \geq 0$

   iii) $\{z \in G : f(z) = 0\}$ has a limit point in G. **8**

   b) State and prove Liouville's theorem. **6**

   c) State Maximum Modulus Theorem. **2**

5. a) Let G be an open subset of the plane and $f : G \to \mathbb{C}$ an analytic function. If γ is a closed rectifiable curve in G such that $\eta(r; w) = 0$ for all W in $\mathbb{C} - G$ then prove that for a in $G - \{\gamma\}$

$$\eta(\gamma; a)f(a) = \frac{1}{2\pi i}\int_{\gamma}\frac{f(z)}{z-a}\,dz\,.$$ **8**

   b) Let f be analytic on D = B(0; 1) and suppose $|f(z)| \leq 1$ for $|z|<1$. Show $|f'(0)| \leq 1$. **4**

   c) Evaluate $\int_{\gamma}\frac{e^z - e^{-z}}{z^n}\,dz$ where n is a positive integer and $\gamma(t) = e^{it}\ 0 \leq t \leq 2\pi$. **4**

6. a) State and prove Goursat's theorem. **8**

   b) If G is simply connected and $f : G \to \mathbb{C}$ is analytic in G then prove that f has a primitive in G. **6**

   c) State Fundamental Theorem of Algebra. **2**

7. a) State and prove Rouche's Theorem. **6**

   b) If G is a region with a in G and if f is analytic on G – {a} with a pole at z = a. In prove that there is a positive integer m and an analytic function $g : G \to \mathbb{C}$ such that

$$f(z) = \frac{g(z)}{(z-a)^m}\,.$$ **5**

   c) Show $\int_0^{\infty}\frac{\sin x}{x}\,dx = \frac{\pi}{2}$. **5**

8. a) Let G be a region in $\mathbb{C}$ and f an analytic function on G. Suppose there is a constant M such that $\limsup |f(z)| \leq M \; \forall a$ in $\partial_\infty G$. Then prove that $|f(z)| \leq M \; \forall z$ in G. **6**

b) Let D = {z | |z| < 1} and let $f : D \to D$ be a one-one analytic map of D onto itself and suppose f(a) = 0. Then prove that there is a complex number C with |C| = 1 such that $f = C\phi_a$ where ϕ_a is a one-one map of D onto itself. **6**

c) Let f be analytic in B(a;R) and suppose that f(a) = 0. Show that 'a' is a zero of multiplicity m iff

$f^{(m-1)}(a) = \ldots\ldots = f(a) = 0$ and $f^{(m)}(a) \neq 0$. **4**

Seat No.	

M.A./M.Sc. (Semester – III) Examination, 2012
MATHEMATICS
MT 702 : Ring Theory
(2008 Pattern)

Time : 3 Hours Max. Marks : 80

N.B. : *1) Attempt **any five** questions.*
*2) Figures to the **right** indicate **full** marks.*
*3) **Each** question carry **equal** marks.*

1. a) Prove that a finite ring R is field iff it is integral domain. **6**

b) If R is a ring of all continuous functions from [0, 1] to set of all real numbers $\mathbb{R}$ then : **6**

i) Find units of R

ii) Find a function in R which is neither unit nor a zero-divisor

iii) Is R an integral domain.

c) Prove that the only Boolean ring that is an integral domain is $\frac{Z}{2Z}$. **4**

2. a) If A is a subring and B is an ideal of the ring R then prove that **6**

$$\frac{A+B}{B} \cong \frac{A}{A \cap B}$$

b) Define the centre of a ring.

What is the centre of a division ring ?

If $\phi : R \to S$ is onto homomorphism of rings, then prove that the image of the centre of R is contained in the centre of S. **6**

c) If I and J are ideals of the ring R then prove that ideal IJ is contained in $I \cap J$, under what conditions(s) equality hold ? **4**

3. a) Prove that every ideal in a Euclidean domain is principal. **6**

b) If R is quadratic integer ring $Z\left[\sqrt{-5}\right]$ and N is associated field norm defined by $N\left(a+b\sqrt{-5}\right)=a^2+5b^2$ then show that R is not a Euclidean domain. (w.r.t. this norm) **5**

c) Find a generator for the ideal (85, 1 + 13i) in Z [i]. **5**

4. a) Prove that in a principal ideal domain (non zero) ideal is prime iff it is maximal. **6**

b) Is it true that the quotient of a PID is PID ? Justify.

If not under what condition(s) it is true. (Give proof) **5**

c) If $R = Z\left[\sqrt{-5}\right]$ is a quadratic integer ring then show that ideal $I=\left(2, 1+\sqrt{-5}\right)$ is not principal ideal but I^2 is principal ideal. **5**

5. a) Prove that in principal ideal domain a non-zero element is a prime if and only if it is irreducible. **6**

b) Consider the ring R = Z [2i] = $\left\{a+2bi \mid a, b\in Z\right\}$ show that (i) the elements 2 and 2i in R are irreducible but not associate in R. (ii) 2 i is not prime in R. (iii) Is R a unique factorization domain ? **6**

c) Prove that I = (1 + i) is a maximal ideal in Z [i] and hence show that the quotient ring $\frac{Z[i]}{(1+i)}$ is a field of order 2. **4**

6. a) If I is an ideal of the ring R and (I) = I [x] is an ideal of R[x] generated by I then prove that $\frac{R[x]}{(I)} \simeq \left(\frac{R}{I}\right)[x]$. What happens if I is a prime ideal of R. **6**

b) If R = Q[x, y], polynomial ring in two variables x and y over the rational numbers then prove that :

i) The ideal I = (x) is prime but not maximal in R.

ii) The ideal J = (x, y) is maximal in R.

iii) The ideal J = (x, y) is root principal in R. **6**

c) Describe the ring structure of the following rings : **4**

i) $\frac{Z[x]}{(2)}$ ii) $\frac{Z[x]}{(x)}$.

7. a) If R is a UFD with field of fraction F and if $p(x) \in R[x]$.

Prove that if p (x) is reducible in F [x] then p (x) is reducible in R [x]. **8**

b) If F is a field and R is set of all polynomials in F [x] whose coefficient of x is zero. Show that R is not a UFD. **4**

c) If R is the set of all polynomials in x with rational coefficients whose constant term is an integer then prove that x cannot written as the product of irreducibles in R. **4**

8. a) State and prove Eisenslein's criterion for irreducibility of polynomial. **6**

b) Prove that if F is a finite field then the multiplicative group F* of non-zero elements of F is a cyclic group. **5**

c) Construct the field with 9 elements. **5**

B/I/12/260

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Seat No.	

M.A./M.Sc. (Semester – III) Examination, 2012
MATHEMATICS
MT-705 : Graph Theory
(2008 Pattern)

Time : 3 Hours Max. Marks : 80

N.B. : *1) Attempt* ***any five*** *questions.*
2) Figures to the ***right*** *indicate* ***full*** *marks.*

1. a) Prove that every closed walk contains an odd cycle in a graph. **6**

   b) Prove that an edge is a cut edge if and only if it belongs to no cycle. **6**

   c) Is an even graph with even number of vertices bipartite ? Justify. **4**

2. a) Prove that a graph is bipartite if and only if it has no odd cycle. **6**

   b) Prove that a complete graph K_n can be expressed as the union of k bipartite graphs if and only if $n \leq 2^k$. **6**

   c) Use Havel-Hakimi theorem to determine whether the sequence (5, 5, 4, 4, 2, 2, 1,1) is graphic. Provide construction or proof of impossibility. **4**

3. a) Prove that a graph is Eulerian if and only if it has atmost one nontrivial component and its vertices all have even degree. **8**

   b) Let G be a graph with n verticcs then prove that the following statements are equivalent. **8**

   A) G is connected and has no cycles.

   B) G is connected and has n – 1 edges.

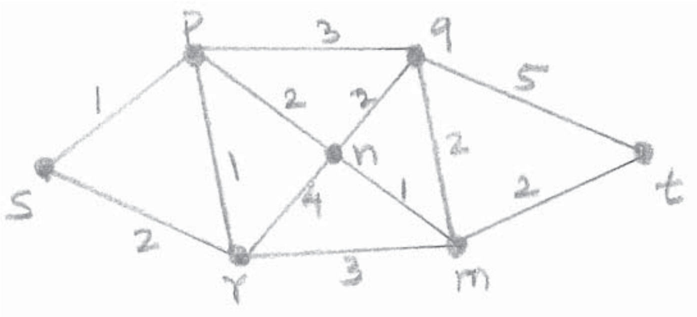
   C) G has n – 1 edges and no cycles).

   D) G has no loops and has, for each u, v $\in$ V (G), exactly u – v path.

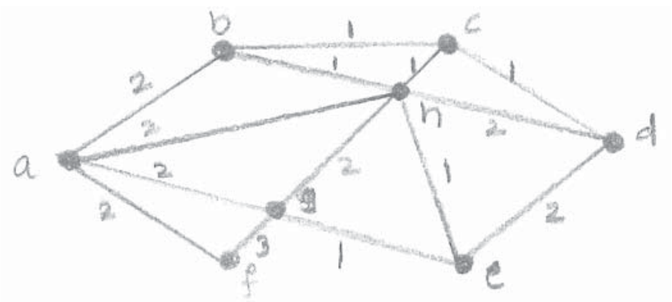
P.T.O.

4. a) Prove that every loopless graph G has a bipartite subgraph with atleast e(G)/ 2 edges **6**

b) Find the shortest path from vertex s to vertex t in the following graph. **6**

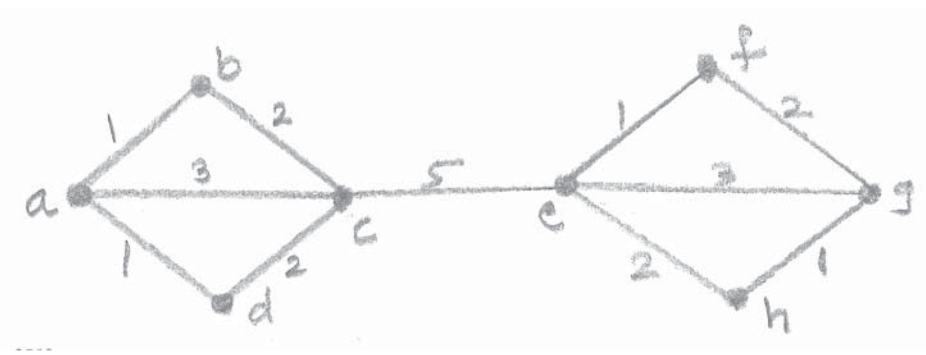


c) Using Kruskal Algorithm find the minimal spanning tree of the following graph. **4**



5. a) Prove that a for a set $S \subset \mathbb{N}$ for size n, there are n^{n-2} trees with vertex set S. **8**

b) Solve the Chinese Postman problem for the following graph. **8**



6. a) Prove that an X, Y - bigraph G has a matching that saturates X if and only if $|N(S)| \geq |S|$ for all $S \subset X$. **8**

   b) Prove that if G is a graph without isolated vertices, then $\alpha'(G) + \beta'(G) = n(G)$. **8**

7. a) Prove that the Hungerian Algorithm finds a maximum weight matching and a minimum cost cover. **6**

   b) If G is a simple graph, then prove that $k(G) \leq k'(G) \leq \delta(G)$. **6**

   c) Define a tournament and a king in a digraph. Prove that every tournament has a king. **4**

8. a) If a graph G has degree sequence $d_1 \geq d_2 \geq \ldots \geq d_n$, then prove that $X(G) \leq 1 + \max_i \min \{d_i, i - 1\}$. **6**

   b) Let $G \otimes H$ denote the cartesian product of two graphs G and H. Prove that $X(G \otimes H) = \max \{_X(G),_X(H)\}$. **6**

   c) Draw a graph whose vertex connectivity is 4, edge connectivity is 5, and the minimum degree of a vertex is 6. **4**

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Seat No.	

M.A./M.Sc. (Semester – I) Examination, 2012
MATHEMATICS
MT-501 : Real Analysis – I
(2008 Pattern)

Time : 3 Hours Max. Marks : 80

N.B. : *1) Attempt* ***any five*** *questions.*
2) Figures to the ***right*** *indicate* ***full*** *marks.*

1. a) If (V, < ., .>)is an complex inner product space and || V || is defined by $\| v\| = \sqrt{< v,v >}$ then show that || . || is a norm on V. **6**

b) Show that d (x, y) = $\dfrac{|x-y|}{1+|x-y|}$ defines a metric on (0, ∞). **5**

c) Verify that l' is normal linear space. **5**

2. a) State and prove Heine-Borel Theorem. **8**

b) If E is a compact subset of a metric space then prove that it is closed. **6**

c) Give an example to show that arbitrary intersection of open set in a metric space is not open. **2**

3. a) Show that $C([a, b], \mathbb{R})$ with supremum norm is complete. **6**

b) Prove that a totally bounded set is bounded. Is the converse true ? Justify. **6**

c) Show that $\mathbb{R}$ with the discrete metric is not separable. **4**

4. a) For an interval I = $[a_1, b_1]$ X ... X $[a_n, b_n]$ in $\mathbb{R}^n$ define m(I) = $\underset{k=1}{\overset{n}{\pi}}(b_k - a_k)$

Let $\in$ is the collection of all finite unions of disjoint intervals in $\mathbb{R}^n$. Show that m is a measure on $\in$. **6**

b) Let A be any subset of $\mathbb{R}^n$ and let $\{I_k\}$ be countable covering of A. Prove that the function m*(A) defined by m* (A) = inf $\sum_{k=1}^{\infty} M(I_k)$ is countable sub additive. **5**

c) Prove that if f is a measurable function then $|f|$ is measurable. Give an counter example to show that if |f| is measurable f may not be measurable. **5**

P.T.O.

5. a) State and prove Lebesgue Monotone convergence theorem. **8**

b) State and prove Hölder's Inequality. **6**

c) State Fatou's Lemma. **2**

6. a) Suppose that $f = \sum_{k=1}^{\infty} c_k f_k$ for an orthonormal sequence $\{f_k\}_{k=1}^{\infty}$ in an inner product space V . Then show that $C_k = < f, f_k >$ for each k. **6**

b) For f and g in an inner product space, $g \neq 0$. show that the two vector

$\frac{< f, g >}{\| g \|^2}$ and $\frac{f - < f, g > g}{\| g \|^2}$ are orthogonal. **5**

c) Show that the trigonometric system $\frac{1}{\sqrt{2\pi}}, \frac{\text{Cos}(nx)}{\sqrt{\pi}}, \frac{\text{Sin}(mx)}{\sqrt{\pi}}$ n, m = 1, 2 ... is an orthonormal sequence in $L^2([-\pi, \pi], m)$. **5**

7. a) State and prove Bessel's Inequality. **6**

b) Give an example of a sequence of functions which is pointwise convergent but not uniformly. Justify. **5**

c) Show that the classical Fourier series of f(x) = x is $2\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ Sin (n x). **5**

8. a) State and prove Cauchy -Schwarz inequality. **6**

b) Show that a Riemann integrable function is also Lebesgue integrable. **5**

c) If f and g are measurable function then show that fg is measurable. **5**

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Seat No.	

M.A./M.Sc. (Semester – I) Examination, 2012
MATHEMATICS
MT – 503 : Linear Algebra
(2008 Pattern)

Time : 3 Hours   Max. Marks : 80

Instructions : *1) Attempt **any five** questions.*
2) Figures to the right indicate maximum marks.

1. a) Let V and V' be finite dimensional vector spaces over K of dimensions n and m respectively. Prove that dimL(V, V') = nm. **6**

   b) Let V be a finite dimensional vector space over K and let W be a subspace of V. Prove that dimV = dimW + dim V/W. **6**

   c) Let $I = (-a, a)$, $a > 0$ be an open interval in R and let $V = R^I$, the space of all real valued functions defined on I. Show that $V = V_e \oplus V_0$, where V_e is the set of all even functions on I and V_0 is the set of all odd functions on I. **4**

2. a) Let B be an ordered basis of an n-dimensional vector space V over K. If T is a linear operator on V, then prove that T is a bijection if and only if $[T]_B$ is an invertible matrix. **6**

   b) Let $V_1, V_2, ..., V_m$ be vector spaces over a field K. Prove that $V = V_1 \oplus ... \oplus V_m$ is finite dimensional if and only if each V_i is finite dimensional. Also prove that $\dim V_1 \oplus ... \oplus V_m = \dim V_1 + ... + \dim V_m$. **6**

   c) Consider the vector space $R_3[x]$ of polynomials with real coefficients and of degree at most 3. The differential operator D is a linear operator on $R_3[x]$. Write the matrix representation of D with respect to $B_1 = \{1 + x, x + x^2, x^2 + x^3, x + x^3\}$. **4**

3. a) Let $A \in K^{n\times n}$. The left multiplication by A defines a linear operator $\lambda_A : K^{n\times m} \to K^{n\times m}$ such that $\lambda_A(B) = AB$. Prove that α is an eigenvalue of λ_A if and only if α is an eigenvalue of A. **6**

   b) Let V and W be finite dimensional vector spaces over K, and let $T \in L(V, W)$. Prove that i) $\ker T^{\bullet} = (\mathrm{im} T)^{\circ}$, ii) $\mathrm{im} T^{\bullet} = (\ker T)^{\circ}$ and iii) rankT = rank$T^{\bullet}$. **6**

c) It T is an invertible linear operator on a finite dimensional vector space over a field K, then prove that the minimal polynomial of T^{-1} is $m_T(0)^{-1} x^r m_T\left(\frac{1}{x}\right)$, where $r = \deg m_T(x)$. **4**

4. a) State and prove the primary decomposition theorem. **10**

b) Prove that the geometric multiplicity of an eigenvalue of a linear operator can not exceed its algebraic multiplicity. **6**

5. a) Let V be a finite dimensional vector space over K of dimension n and let T be a linear operator on V. If the characteristics polynomial of T splits over K, then prove that T is triangulable. **8**

b) Prove that a Jordan chain consists of linearly independent vectors. **4**

c) The characteristics polynomial of a matrix is $(x-1)^3 (x-2)^2$. Write its Jordan canonical forma. **4**

6. a) Give all possible rational canonical forms it the characteristic polynomial is :

i) $(x^2 + 2)(x - 3)^2$;

ii) $(x - 1)^2 (x + 1)^2$. **6**

b) Let V be a finite dimensional vector space over K and let T be a linear operator on V. Prove that V is a direct sum of T – cyclic subspaces. **10**

7. a) Prove the polarization identities for the inner product space. **4**

b) Let V be a finite dimentional inner product space and let f be a linear functional on V. Prove that there exists a unique vector x in V such that f(v) = (v,x), for all v in V. **8**

c) Let T be triangulable linear operator on an n-dimensional inner product space V and let all the eigenvalues of T are equal to 1 in absolute value. If $\|Tv\| \leq \|v\|$ for all $v \in V$, then show that T is unitary. **4**

8. a) Let T be a self adjoint operator on an inner product space V. Prove that all roots of characteristic polynomial of T are real. **5**

b) Consider the inner product space $R_3[x]$ with the inner product

$(p(x), q(x)) = \int_{-1}^{1} p(x)q(x)dx$. Find the adjoint of the differential operator D. **5**

c) Find a polar decomposition of the following matrix. $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. **6**

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Seat No.	

M.A./M.Sc. (Semester – I) Examination, 2012
MATHEMATICS
MT-504 : Number Theory
(2008 Pattern)

Time : 3 Hours Max. Marks : 80

Instructions : *1) Attempt* ***any five*** *questions.*
2) Figures to the ***right*** *indicate* ***full*** *marks.*

1. a) Let a, b be integers, b > 0. Show that there exist unique integers q and r such that
$a = bq + r,\ 0 \le r < b.$ **6**

b) Find the highest power of 15 that divides 1000 ! **4**

c) Show that there are infinitely many primes in the arithmetic progression 3n + 1. **6**

2. a) Let p be a prime. Show that the congruence $x^2 \equiv -1 \pmod{m}$ has a solution if and only if p = 2 or $p \equiv 1 \pmod{4}$. **6**

b) Show that gcd $(2^{2^m}+1, 2^{2^n}+1) = 1$ if $m \neq n$. **5**

c) Find the smallest positive integer N such that when divided by 7 leaves remainder 3, when divided by 33 leaves remainder 32 and when divided by 13 leaves remainder 10. **5**

3. a) Let p be an odd prime and gcd (a, p) = 1. Consider the integers
a, 2a, ..., $\{(p-1)/2\}a$ and their least non-negative residues modulo p. If n denotes the
number of residues modulo p then $\left(\dfrac{a}{p}\right) = (-1)^n$. **6**

b) Decide whether the congruence $x^2 \equiv -42 \pmod{61}$ has a solution. **4**

c) Prove that if n is an integer then $504 \mid n^9 - n^3$. **6**

P.T.O.

4. a) If p is a prime $p \equiv 1 \pmod 4$ then show that there exist integers a, b such that $p = a^2 + b^2$. **10**

b) Find all integers x and y such that $147x + 258y = 369$. **6**

5. a) Define a multiplicative function. If f(n) is multiplicative, then prove that $F(n) = \sum_{d/n} f(d)$ is a multiplicative function. **6**

b) If gcd(m, n) = 1 then prove that $\phi(mn) = \phi(m)\phi(n)$. **5**

c) Prove that 3 is a prime in $\mathbb{Q}(i)$ but not a prime in $\mathbb{Q}(\sqrt{6})$. **5**

6. a) If α and β are algebraic numbers then show that $\alpha + \beta, \alpha\beta$ are algebraic numbers. Further, show that if α and β are algebraic integers then show that $\alpha + \beta, \alpha\beta$ are algebraic integers. **8**

b) Show that if p is prime then $\binom{p}{k} \equiv 0$ (mod p) for $1 \leq k \leq p - 1$. **4**

c) Let n be a positive integer. Show that d(n) is odd if and only if n is a perfect square. **4**

7. a) Let m be a negative square-free national integer. Determine all the units in the field $\mathbb{Q}(\sqrt{m})$. **7**

b) If α is an algebraic number, then prove that there exist an integer b such that $b\alpha$ is an algebraic integer. **4**

c) Prove that the field $\mathbb{Q}(\sqrt{-14})$ does not have unique factorization property. **5**

8. a) Using unique factorization property of $\mathbb{Q}[i]$ or otherwise, determine all solutions of $x^2 + y^2 = z^2$ in positive integers such that gcd (x, y, z) = 1. **6**

b) Find the minimal polynomial of $1 + \sqrt{2} + \sqrt{3}$. **5**

c) Prove that if p and q are distinct primes of the form 4k + 3, and if $x^2 \equiv p \pmod q$ has no solution, then $x^2 \equiv q \pmod p$ has two solutions. **5**

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Seat No.	

M.A./M.Sc. (Semester – II) Examination, 2012

MATHEMATICS

MT-601 : General Topology

(New) (2008 Pattern)

Time : 3 Hours | Max. Marks : 80

N.B. : *i) Attempt* ***any five*** *questions.*

ii) Figures to the ***right*** *indicate* ***full*** *marks.*

1. a) Let X be any non-empty set, let τ_c be a collection of all subsets U of X such that X – U is either is finite or all of X. Then show that τ_c is a topology on X. **5**

b) Let $\mathscr{B}$ and $\mathscr{B}'$ be bases for the topologies τ & τ' respectively on X then show that τ' is finer than τ iff for each $x \in X$ and each basis element $B \in \mathscr{B}$ containing x there is a basis element $B' \in \mathscr{B}'$ such that $x \in B' \subset B$. **5**

c) Show that the countable collection.

$\mathscr{B} = \{(a, b) \mid a \,\&\, b$ are rational $\}$ is a basis that generates the standard topology on $\mathbb{R}$. **6**

P.T.O.

2. a) Show that the collection

$S = \{\pi_1^{-1}(U) \mid U \text{ open in } X\} \cup \{\pi_2^{-1}(V) \mid V \text{ open in } Y\}$ is a subbasis for the product topology on $X_x Y$. **5**

b) If A is a subspace of X and B is a subspace of Y then show that the product topology on A×B is the same as the topology on A×B inherits as a subspace of X×Y. **6**

c) Let X be a topological space let $A \subseteq X$ if $\tau_A = \{U \subseteq A \mid U = V \cap A,$ for some V open in $X\}$ then show that τ_A is a topology on A. **5**

3. a) Let Y be a subspace of X : Let $A \subseteq Y$, let $\overline{A}$ denote the closure of A in X. then show that closure of A in Y equals $\overline{A} \cap Y$. **4**

b) Show that every order topology is Hausdorff. **4**

c) Show that Int A and Bd A are disjoint and $\overline{A} = \text{Int } A \cup \text{Bd } A$. **4**

d) State and prove the pasting lemma. **4**

4. a) Show that [0, 1] & [a, b] are homeomorphic. **6**

b) Let $f : A \to X_x Y$ be given by the equation $f(a) = (f_1(a), f_2(a))$. Then show that f is continuous iff the functions $f_1 : A \to X$ and $f_2 : A \to Y$ are continuous. **5**

c) Let $f : A \to \pi_{\alpha \in J} X_\alpha$ be given by the equation $f(a) = (f_\alpha(a))_{\alpha \in J}$, where $f_\alpha : A \to X_\alpha$ for each α. Let πx_α have the product topology. Then show that the function f is continuous iff each function f_α is continuous. **5**

5. a) Show that the topologies on $\mathbb{R}^n$ induced by the Euelidean metric d and the square metric ρ are the same as the product topology on $\mathbb{R}^n$. **6**

b) Let X be a topological space; Let $A \subset X$. If there is a sequence of points of A converging to x, then show that $x \in \overline{A}$, the converse is true if X is metrizable. **5**

c) Prove that retraction map is a quotient map. **5**

6. a) Let $g : X \to \mathbb{Z}$ be a surjective continuous map Let X* = {$g^{-1}(z)/z \in \mathbb{Z}$}. Give X* the quotient topology show that the map g induces a bijective continuous map f : X* $\to$ z, which is a homeomorphism iff g is a quotient map. **5**

b) Let $\{A_\alpha\}$ be collection of connected subspaces of X. Let A be a connected subspace of X. Show that if $A \cap A_\alpha \neq \phi \forall \alpha$, then $A \cup \left(\bigcup_\alpha A\alpha\right)$ is connected. **5**

c) Show that a path connected space X is connected. Is converse is true ? Justify. **6**

7. a) Let $\{A_\alpha\}$ is collection of path connected subspaces of X and if $\cap A_\alpha \neq \phi$, is $\bigcup_\alpha A_\alpha$ necessarily path connected ? **6**

b) Show that every closed subspace of compact splace is compact. **5**

c) Let X be locally compact Hausdorff; let A be a subspace of X. If A is closed in X or open in X then show that A is locally compact. **5**

8. a) Show that a connected metric space having more than one elements is uncountable. **5**

b) Show that every metricable space is normal. **5**

c) Show that a subspace of completely regular space is completely regular. **4**

d) State Tychonoff theorem. **2**

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Seat No.	

M.A./M.Sc. (Semester – II) Examination, 2012

MATHEMATICS

MT-601 : Real Analysis – II (Old)

Time : 3 Hours Max. Marks : 80

***N.B.** : i) Attempt **any five** questions.*

*ii) Figures to the **right** indicate **full** marks.*

1. a) With usual notations, prove that $\| f_1 f_2 \|_{BV} \leq \| f_1 \|_{BV} \| f_2 \|_{BV}$. **5**

   b) If $f \in R_\alpha[a, b]$, then prove that $\alpha \in R_f[a, b]$ and

   $\int_a^b f d\alpha + \int_a^b \alpha df = f(b)\alpha(b) - f(a)\alpha(a)$. **6**

   c) True or False. Justify.

   A bounded continuous function is of bounded variation. **5**

2. a) If f is Riemann integrable on [a, b], then prove that f is Lebesgue integrable. **6**

   b) Let $f \in R_\alpha[a, b]$ and c be a real number. Prove that

   $cf \in R_\alpha[a,b]$ and $\int_a^b cf\, d\alpha = c\int_a^b f\, d\alpha$ **5**

   c) Give example of a bounded function which is not Riemann integrable. Justify your answer **5**

3. a) Let f, g$\in$ BV [a, b] and a$\leq$ c$\leq$ b, then prove that

$$V_a^b(f+g) \leq V_a^b(f) + V_a^b(g) \text{ and}$$

$$V_a^b(f) = V_a^c(f) + V_c^b(f).$$ **6**

b) Write the Fourier series for the following function : **6**

$$f(x) = x \text{ for } x \in [-\pi, \pi]$$

c) Define the outer measure. Give example of a set with outer measure zero. Justify **4**

4. a) If E and F are disjoint compact sets, then prove that

$$m^*(E \cup F) = m^*(E) + m^*(F).$$ **6**

b) Show that the improper Riemann integral $\int_0^\infty \frac{\sin x}{x} dx$ exists. **6**

c) Let $\{f_n\}$ be a sequence of measurable functions, then show that $\sup_n f_n$ is measurable. **4**

5. a) Show that the sum of two measurable functions is measurable. **6**

b) Suppose f is a non-negative and measurable function, then show that $\int f = 0$ if and only if f = 0 a.e. **5**

c) If F is a closed subset of a bounded open set G, then prove that m*(G/F)=m*(G) – m*(F) **5**

6. a) Let $1 < p < \infty$ and q be defined by $\frac{1}{p} + \frac{1}{q} = 1$. If $f \in L_p(E)$ and $g \in L_q(E)$, then prove that $fg \in L_1(E)$ and $\left|\int_E fg\right| \leq \int_E |fg| \leq \|f\|_p \|g\|_q$. **5**

   b) Let $\{E_n\}$ be a sequence of measurable sets. If $E_n \supset E_{n+1}$ for each n and $m(E_k)$ is finite for some k, then prove that

$$m\left(\bigcap_{n=1}^{\infty} E_n\right) = \lim_{n\to\infty} m(E_n)$$ **6**

   c) Show that $\lim_{n\to\infty} \int_0^1 f_n = 0$ where $f_n(x) = \frac{nx}{1+n^2x^2}$. **5**

7. a) State and prove Fatou's Lemma. **8**

   b) If f is a measurable function, then show that |f| is measurable. Is the converse true ? Justify. **5**

   c) Give an example of absolutely continuous function. **3**

8. a) State and prove Lebesgue's Dominated Convergence theorem. **8**

   b) Give an example of a non-measurable set. **8**

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Seat No.	

M.A./M.Sc. (Semester – II) Examination, 2012
MATHEMATICS
MT-602 : Differential Geometry
(2008 Pattern)

Time : 3 Hours Max. Marks : 80

N.B. : *i) Attempt **any five** questions.*
*ii) Figures to the **right** indicate **full** marks.*

1. a) Let U be an open subset of R^{n+1} and $f : U \rightarrow R$ be a smooth function. Let $p \in U$ be a regular point of f and let $c = f(p)$. Show that the set of all vectors tangent to $f^{-1}(c)$ at p is equal to $[\nabla f(p)]^{\perp}$. **6**

b) Find the integral curve of the vector field X given by $X(x_1, x_2) = (x_1, x_2, x_2, -x_1)$ through the point (1, 1). **5**

c) Show that the graph of any smooth function $f : R^n \rightarrow R$ is an n-surface in R^{n+1}. **5**

2. a) Let S be a connected n-surface in R^{n+1}. Show that on S, there exists exactly two smooth unit normal vector fields N_1 and N_2. **6**

b) Sketch the following vector fields on R^2 : $X(p) = (p, X(p))$ where

i) $X(p) = -p$

ii) $X(x_1, x_2) = (x_2, x_1)$. **4**

c) Let $a, b, c, d \in R$ be such that $ac - b^2 > 0$. Show that the maximum and minimum values of the function $g(x_1, x_2) = x_1^2 + x_2^2$ on the ellipse $ax_1^2 + 2bx_1x_2 + cx_2^2 = 1$ are of the form $\frac{1}{\lambda_1}$ and $\frac{1}{\lambda_2}$ where λ_1 and λ_2 are eigen values of the matrix $\begin{pmatrix} a & b \\ b & c \end{pmatrix}$. **6**

3. a) Let U be an open subset of R^{n+1} and $f : U \to R$ be a smooth function. Let $S = f^{-1}(c)$, $c \in R$ and $\nabla f(q) \neq 0$, $\forall q \in S$. If $g : U \to R$ is smooth function and $p \in S$ is an extreme point of g on S, then show that there exists a real number λ such that $\nabla g(p) = \lambda \nabla f(p)$. **6**

   b) Show that the tangent space to $SL_2(R)$ at $P = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ can be identified with the set of all 2×2 matrices of trace zero. **6**

   c) Show that the speed of geodesic is constant. **4**

4. a) Show that the covariant differentiation has the following property : $(X.Y)' = X'.Y + X.Y'$. **5**

   b) Consider a vector field $X(x_1, x_2) = (x_1, x_2, 1, 0)$ on R^2. For $t \in R$ and $p \in R^2$, let $\phi_t(p) = \alpha_p(t)$ where α is the maximal integral curve of X through p. Show that $F(t) = \phi_t$ is a homomorphism of additive group of real numbers into the invertible linear maps of the plane. **6**

   c) Show that the Weingarten map of the n-sphere of radius r oriented by inward normal is multiplication by $\frac{1}{r}$. **5**

5. a) Let $\alpha(t) = (x(t), y(t))$ be a local parametrization of the oriented plane curve C. Show that $k \circ \alpha = \dfrac{x'y'' - y'x''}{(x'^2 + y'^2)^{\frac{3}{2}}}$. **5**

   b) Find the curvature of the circle with centre (a, b) and radius r oriented by the outward normal. **5**

   c) Show that the Weingarten map L_p is self-adjoint.

   (that is $L_p(v).\, w = v.\, L_p(w)$, for all $v, w \in S_p$). **6**

6. a) Let S be an n-surface in R^{n+1}, let $\alpha : I \to S$ be a parametrized curve in S, let $t_0 \in I$ and $v \in S_{\alpha(t_0)}$. Prove that there exists a unique vector field V tangent to S along α, which is parallel and has $V(t_0) = v$. **6**

b) Let S denote the cylinder $x_1^2 + x_2^2 = r^2$ of radius r in R^3. Show that α is a geodesic of S if and only if α is of the form $\alpha(t) = (r\cos(at + b), r\sin(at + b), ct + d)$ for some real numbers a, b, c, d. **6**

c) Define the Gauss map and the spherical image of the oriented n -surface S. **4**

7. a) Prove that on each compact oriented n-surface S in R^{n+1} there exists a point p such that the second fundamental form at p is definite. **6**

b) Let C be a connected oriented plane curve and let $\beta : I \to C$ be a unit speed global parameterization of C. Show that β is either one to one or periodic. **5**

c) Show that the 1-form η on $R^2 - \{0\}$ defined by

$$\eta = \frac{-x_2}{x_1^2 + x_2^2} dx_1 + \frac{x_1}{x_1^2 + x_2^2} dx_2$$ is not exact. **5**

8. a) Let S be an n-surface in R^{n+1} and let $p \in S$. Prove that there exists an open set V about p in R^{n+1} and a parametrized n-surface $\phi : U \to R^{n+1}$ such that ϕ is one to one map from U onto $V \cap S$. **8**

b) Let S be the ellipsoid $\frac{x_1^2}{a^2} + \frac{x_2^2}{b2} + \frac{x_3^2}{c^2} = 1$, a, b, c, all non -zero , oriented by the outward normal. Show that the Gaussian curvature of S is

$$K(p) = \frac{1}{a^2b^2c^2\left(\frac{x_1^2}{a^4} + \frac{x_2^2}{b^4} + \frac{x_3^2}{c^4}\right)^2}.$$ **8**

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Seat No.	

M.A./M.Sc. (Semester – II) Examination, 2012
MATHEMATICS
MT-606 : Object Oriented Programming with C++
(2008 Pattern)

Time : 2 Hours   Max. Marks : 50

N.B. : *i) Question* ***1*** *is* ***compulsory****.*
ii) Attempt any ***2*** *out of question* ***2, 3*** *and* ***4****.*
iii) Figures at ***right*** *indicate* ***full*** *marks.*

1. Attempt the following questions : **20**

   i) What are drawbacks of procedure oriented programming languages ?

   ii) Write output of following program

```
# include <iostream.h>
   int main ( )
   {
     cout << "Mathematics is bueatifull";
     return o;
   }
```

   iii) Write content and purpose of header file <float.h>.

   iv) State 6 relational operators used in C++.

   v) State one difference between break and continue.

   vi) How does class achieve data hiding ?

   vii) Write general syntax for function declaration in C++.

   viii) Write a program in C++ to find square of a number.

   ix) Define friend function.

   x) List the operand that cannot be overloaded by C++.

2. i) Write an object oriented program in C++ to multiply two matrices. Let M_1 and M_2 be two matrices. Find out $M_3 = M_1 * M_2$. **10**

ii) Write a note on overloading decrement operator. **5**

3. i) What is difference between constructor and destructer ? **6**

ii) Write a C++ program to find GCD of two positive integer using function. **9**

4. i) Define : **9**

   i) Call by value

   ii) Call by reference

   iii) Return by reference with examples.

ii) State the difference between if else statement and switch statement. **6**

B/I/12/200

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Seat No.	

M.A./M.Sc. (Semester – III) Examination, 2012
MATHEMATICS
MT – 701 : Functional Analysis
(2008 Pattern)

Time : 3 Hours Max. Marks : 80

*N.B. : i) Attempt **any five** questions.*
*ii) Figures to the **right** indicate **full** marks.*

1. a) State and prove Hahn-Banach theorem. **8**

   b) Show that $\| T^* \| = \| T \|$ and $\| T^*T \| = \| T \|^2$. **6**

   c) A linear operator $T : l^2 \rightarrow l^2$ is defined by
   $T(x_1, x_2, ...) = \left(x_1, \frac{x_2}{2}, ..., \frac{x_n}{n}, ... \right)$. Find its adjoint T*. **2**

2. a) If P is a projection on a Hilbert space H with range M and null space N, show that $M \perp N$ if and only if P is self-adjoint. In this case also show that $N = M^{\perp}$. **6**

   b) Let M be a closed linear subspace of a normed linear space N. If a norm of a coset x+M in the quotient space N/M is defined by $||| x + M ||| = \inf \{ \| x + m \| : m \in M \}$, then prove that N/M is a normed linear space. Further if N is Banach, then prove that N/M is also a Banach space. **8**

   c) Write example of a normal operator. **2**

3. a) Show that an operator T on a finite dimensional Hilbert space H is normal if and only if its adjoint T* is a polynomial in T. **6**

   b) If T is any operator on a Hilbert space H then show that the following conditions are equivalent :

   i) $T^* T = I$

   ii) $\langle Tx, Ty \rangle = \langle x, y \rangle$ for all $x, y \in H$

   iii) $\| T \| = \| x \|$ for all $x \in H$. **6**

P.T.O.

c) If T is any operator on a Hilbert space H and if α, β are scalars such that $|\alpha| = |\beta|$, then show that $\alpha T + \beta T^*$ is normal. **4**

4. a) Give examples of two non-equivalent norms. Justify. **6**

b) Let X and Y be normed spaces. If X is finite dimensional, then show that every linear transformation from X to Y is continuous. Give an example of a discontinuous linear transformation. **8**

c) Show that the norm of an isometry is 1. **2**

5. a) If T is an operator on a Hilbert space H, then prove that T is normal if and only
if its real and imaginary parts commute. **6**

b) Let M be a closed linear subspace of a normed linear space N and T be the natural mapping of N onto N/M defined by T(x) = x + M. Show that T is a continuous linear transformation for which $\|T\| \leq 1$. **6**

c) Show that the unitary operators on a Hilbert space H form a group. **4**

6. a) Let T be an operator on H. If T is non-singular, then show that $\lambda \in \sigma(T)$ if and only if $\lambda^{-1} \in \sigma\left(T^{-1}\right)$. **4**

b) If T is an operator on a Hilbert space H for which $\langle Tx, x \rangle = 0$ for all x $\in H$, Rthen prove that T = 0. **6**

c) Let S and T be normal operators on a Hilbert space H. If S commutes with T*, then prove that S + T and ST are normal. **6**

7. a) State and prove the Closed Graph Theorem. **8**

b) Let T be a normal operator on H with spectrum $\{\lambda_1, \lambda_2, \dots \lambda_m\}$. Show that T is self-adjoint if and only if each λ_i is real. **4**

c) Find $M^{\perp}$ if M = { (x, y) : x + y = 0} $\subset R^2$. **4**

8. a) Let H be a Hilbert space and f be a functional on H. Prove that there exists a unique vector y in H such that $f(x) = \langle x, y \rangle$ for every $x \in H$. **8**

b) Prove that every finite dimensional subspace of a normed linear space X is closed. Give an example to show that an infinite dimensional subspace of a normed linear space may not be closed. **8**

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B/I/12/260

Seat No.	

M.A./M.Sc. (Semester – III) Examination, 2012
MATHEMATICS
MT 703 : Mechanics (2008 Pattern)

Time : 3 Hours Max. Marks : 80

Instructions : *i) Attempt* ***any five*** *questions.*
ii) ***All*** *questions carry* ***equal*** *marks.*
iii) Figures to the ***right*** *indicate maximum marks.*

1. a) Explain the concept of virtual work and the D'Alember't principle. **4**

b) Define a cyclic cordinate and show that the generalised momentum conjugate to a cyclic coordinate is conserved. **4**

c) State Hamilton's principle and derive Lagrange's equations of motion, from Hamilton's principle. **8**

2. a) Set up Lagrangian for Atwood's machine and write Lagrange's equations of motion. **6**

b) A particle of mass m moves in one dimension such that it has the Lagrangian

$L(x,\dot{x}) = \frac{m}{2}\left(a\dot{x}^2 + 2b\dot{x}\dot{y} + c\dot{y}^2\right) - \frac{K}{2}\left(ax^2 - 2bxy + cy^2\right)$, m, k, a, b and c are constants and $b^2 - ac \neq 0.$ Find the Lagrange's equations of motion and find the solution. **6**

c) Determine the number of degrees of freedom in case of (i) conical pendulum (ii) a free particle moving in a plane. **4**

3. a) Under which conditions H = T + V, where the symbols have the standard meaning ? **4**

b) Find the values of α and β for which the following equations

$Q = q^{\alpha} \cos\beta p, P = q^{\alpha} \sin\beta p$

represent a canonical transformation. **4**

P.T.O.

c) Show that identity transformation can not be generated by F_1, or F_4 type of functions. **4**

d) Consider motion of a free particle having mass m in a plane. Express its kinetic energy in terms of plane polar coordinates and their time derivatives. **4**

4. a) If the Hamiltonian H of the system is

$$H = \frac{p^2}{2} - \frac{1}{2q^2}$$

then show that $(pq/2 - Ht)$ is a constant of motion. **6**

b) A particle is moving under the force derived from the generalized potential $V = cx\dot{x}$, where c is a constant. Choose x, y, z as generalized co-ordinates and write down the Lagrangian of the particle, and hence obtain its generalized momentum p_x. **4**

c) Find the transformation generated by

$$F_1(q, Q) = qQ \; - m\omega q^2/2 - \; Q^2 \; /(4,mw) \,,$$

where m, w are constants. **6**

5. a) Find the stationary function of the integral

$$\int_{-1}^{1} ((y')^2 - 2xy)dx, \; y(-1) = -1, \; y(1) = 1.$$ **4**

b) Write Hamilton's equations of motion using Poisson brackets. Show that $\frac{dH}{dt} = \frac{\partial H}{\partial t}$, where H denotes Hamiltonian. **4**

c) State and prove the Jacobi identity in case of Poisson brackets. **8**

6. a) State and prove rotation formula. **6**

b) Explain the diagramatically that finite rotations in three dimensions do not commute. **5**

c) Define infinitesimal rotations. Show that infinitesimal rotations are pseudo-vectors. **5**

7. a) State and prove Euler's theorem on the motion of a rigid body. **6**

b) Define orthogonal transformations. Show that orthogonal transformations in two dimensions is equivalent to a rotation of coordinate axes. **5**

c) What are Euler angles ? Explain diagramatically. **5**

8. a) Define central force motion. Show that it is always planar. Further show that the areal velocity is constant. **5**

b) A particle moves along the curve,

$\bar{r} = (\cos t)\hat{i} + (\sin t)\hat{j} + t\hat{k}$,

starting at t = 0. Find velocity and acceleration at $t = \pi/2$. **5**

c) Show that the central force motion of two bodies about their center of mass can always be reduced to an equivalent one body problem. **6**

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Seat No.	

M.A./M.Sc. (Semester – III) Examination, 2012
MATHEMATICS
MT – 704 : Measure and Integration (New)
(2008 Pattern)

Time : 3 Hours Max. Marks : 80

Instructions : *1) Attempt **any five** questions.*
*2) Figures to the **right** indicate **full** marks.*

1. a) If E_i' s are with $\mu E_1 < \infty$ and $E_i \supset E_{i+1}$ then prove that

$$\mu\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n\to\infty} \mu E_n$$ **6**

b) Show that the collection of locally measurable sets is a σ – algebra. **6**

c) Let f be a bounded measurable function defined on the finite interval (a, b) then show that $\lim\limits_{\beta\to\infty} \int_a^b f(x) \sin(\beta x) dx = 0$. **4**

2. a) If μ is complete measure and f is a measurable function, then f = g a.e. implies g is measurable. **4**

b) Let for each α in a dense set D of real numbers there is assigned a set $B_\alpha \in \mathbb{B}$ such that $\mu(B_\alpha - B_\beta) = 0$ for $\alpha < \beta$. Then prove that there is a measurable function f such that $f \leq \alpha$ a.e. on B_α and $f \geq \alpha$ a.e. on $X \sim B_\alpha$. **6**

c) Show that every countable set has Hausdorff dimension zero. **6**

3. a) State and prove Fatou's Lemma. **6**

b) Let f_n be a sequence of nonnegative measurable functions which converge almost everywhere to a function f and $f_n \leq f$ for all n then prove that $\int f = \lim \int f_n$. **6**

c) Show that monotone functions are measurable. **4**

4. a) If f and g are integrable functions and E is a measurable set the shown that **6**
   i) $\int_E (c_1 f + c_2 g) = c_1 \int_E f + c_2 \int_E g$.
   ii) If $|h| \leq |f|$ and h is a measurable then h is integrable.
   iii) If $f \geq g$ a.e. then $\int f \geq \int g$.

b) Let $(X, \mathbb{B})$ be a measurable space, $<\mu_n>$ a sequence of measures that converge set wise to a measure μ and $<f_n>$ a sequence of nonnegative measurable functions that converge pointwise to the function f then show that $\int f d\mu \leq \underline{\lim} \int f_n d\mu_n$. **6**

c) Show that there exist uncountable sets of zero measure. **4**

P.T.O.

5. a) Let v be a signed measure on the measurable space $(X, \mathbb{B})$ then prove that there is a positive set A and a negative set B such that $X = A \bigcup B$ and $A \bigcap B = \phi$. **6**

b) Let μ, ν and λ be σ-finite. Show that the Radon-Nikodym derivative $\left[\frac{d\nu}{d\mu}\right]$ has the following properties : **6**

i) If $\nu \ll \mu$ and f is a nonnegative measurable function, then $\int f d\nu = \int f \left[\frac{d\nu}{d\mu}\right] d\mu$.

ii) $\left[\frac{d(\nu_1 + \nu_2)}{d\mu}\right] = \left[\frac{d\nu_1}{d\mu}\right] + \left[\frac{d\nu_2}{d\mu}\right]$.

iii) If $\nu \ll \mu \ll \lambda$ then $\left[\frac{d\nu}{d\lambda}\right] = \left[\frac{d\nu}{d\mu}\right]\left[\frac{d\mu}{d\lambda}\right]$.

c) Show that the outer measure of an interval equals its length. **4**

6. a) Let $(X, \mathbb{B},, \mu)$ be a finite measure space and g an integrable function such that for some constant M, $\left|\int g\phi d\mu\right| \leq M\|\phi\|_p$ for all simple functions ϕ then show that $g \in L^q$. **8**

b) If μ is a finite Baire measure on the real line, then prove that its commutative distribution function F is a monotone increasing bounded function which is continuous on the right and $\lim_{x \to -\infty} F(x) = 0$. **8**

7. a) i) Define an outer Measure μ^*. **8**

ii) Show that the class $\mathbb{B}$ of μ^* - measurable sets is a σ-algebra.

iii) If $\bar{\mu}$ is μ^* restricted to $\mathbb{B}$, then prove that $\bar{\mu}$ is a complete measure on $\mathbb{B}$.

b) Define Product Measure. Let E be a set in $\mathbb{R}_{\sigma\delta}$ with $\mu \times \nu\ (E) < \infty$ then show that the function g defined by $g(x) = \nu E_x$ is a measurable function of x and $\int g d\mu = \mu \times \nu(E)$. **8**

8. a) If μ^* is a Caratheodory outer measure with respect to Γ then prove that every function in Γ is μ^*-measurable. **8**

b) Let B be a μ^*-measurable set with $\mu^* B < \infty$ then prove that $\mu_* B = \mu^* B$. **4**

c) If $< E_i >$ is any disjoint sequence of sets then show that $\sum_{i=1}^{\infty} \mu_* E_i \leq \mu_*\left(\bigcup_{i=1}^{\infty} E_i\right)$. **4**

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Seat No.	

M.A./M.Sc. (Semester – III) Examination, 2012
MATHEMATICS
MT – 704 : Mathematical Methods – I (Old)

Time : 3 Hours Max. Marks : 80

Instructions : *1) Attempt **any five** questions.*
*2) Figures to the **right** indicate **full** marks.*

1. a) Test for convergence the series $1+\frac{1}{2!}+\frac{1}{3!}+\cdots+\frac{1}{n!}+\cdots$ **6**

   b) Find the interval of convergence of the following power series $\sum_{n=0}^{\infty}\frac{(2x)^n}{3^n}$. **5**

   c) To find the series for (x+1) sinx. **5**

2. a) Solve the boundary value problem
   $y_u(x, t) = a^2 y_{xx}(x, t)$, $0 < X < L$, $t > 0$ subject to conditions $y(0,t) = 0$; $y(1, t) = 0$. **8**

   b) To find the Fourier coefficient with usual notations. **8**

3. a) Expand the periodic function in a sine-cosine Fourier series. **8**

$$f(x)=\begin{cases}0, & -\pi < x < 0\\ 1, & 0 < x < \frac{\pi}{2}\\ 0, & \frac{\pi}{2} < x < \pi\end{cases}$$

   b) Define even and odd functions and sketch the graphs of following functions. **8**
   $f(x) = x^2$ and $f(x) = f(x) = \cos x$.

4. a) Prove that $T = 4\sqrt{\frac{l}{2g}}\int_0^{\pi/2}\frac{d\theta}{\sqrt{\cos\theta}}$, where T is period and l is length of simple pendulum. **8**

   b) Prove that $\int_{-1}^{1}P_m(x)\,P_n(x)\,dx = 0$ if $m \neq n$. **8**

5. a) Find P_0 (x), P_1(x), P_2(x), P_3(x) and P_4(x) by using Rodrigue's formula. **10**

b) Prove that $\int_{-1}^{1} [Pn(x)]^2 dx = \frac{2}{2n+1}$ if $m = n$. **6**

6. a) Use Laplace transformation to solve the differential equation $y'' + 4y' + 13y = 20e^{-t}$, subject to conditions $y_0=1$, $y'_0=3$. **8**

b) Find the value of $L^{-1}\left\{\frac{3s+1}{(s+1)^4}\right\}$. **4**

c) Find the Laplace transform of F(t), where $F(t) = \begin{cases} \cos\left(t - \frac{2\pi}{3}\right), & \text{if } t > \frac{2\pi}{3} \\ 0, & \text{if } t < \frac{2\pi}{3} \end{cases}$ **4**

7. a) Obtain the Rodrigues formula for the Lagurre polynomials $L_n(\alpha)$ (X). **8**

b) Expand the following function in Legendre series.

$$f(x) = \begin{cases} 0, & -1 < x < 0 \\ 1, & 0 < x < 1 \end{cases}$$ **8**

8. a) Prove that $J_p(x) J'_{-p}(x) - J_{-p}(x) J'_p(x) = -\frac{2}{\pi x} \sin p\pi$. **8**

b) Show that $L\{t^h f(t)\} = (t)^n \frac{d^n}{ds^n} f(s)$. **8**

Seat No.	

M.A./M.Sc. (Semester – III) Examination, 2012
MATHEMATICS
MT-706 : Numerical Analysis (Old)

Time : 3 Hours Max. Marks : 80

N.B. : *1) Answer* ***any five*** *questions.*
2) Figures to the ***right*** *indicate* ***full*** *marks.*
3) ***Use*** *of unprogrammable, scientific calculator is* ***allowed.***

1. A) Assume that g (x) and $g^1(x)$ are continuous on a balanced interval (a, b) = (P – δ, P + δ) that contain the unique fixed point P and that starting value P_0 is chosen in the interval. Prove that if $\left|g^1(x)\right| \leq K < 1 \forall x \in [a,b]$ then the iteration $P_n = g(P_{n-1})$ coverages to P and if $\left|g^1(x)\right| \succ 1 \forall x \in [a,b]$ then the iteration $P_n = g(P_{n-1})$ does not converges to P. **6**

B) Investigate the nature of iteration in part (A) when $g(x) = -4 + 4x - \frac{x^2}{2}$

i) Show that P = 2 and P = 4 are the fixed points.

ii) Use $P_0 = 1.9$ and compute p_1, p_2, p_3. **5**

C) Start with the interval [3.2, 4.0] and use the Bisection method to find an interval of width h = 0.05 that contain a solution of the equation log (x) – 5 + x = 0. **5**

2. A) Assume that $f \in c^2[a,b]$ and exist number $p \in [a,b]$ where f (p) = 0. If $f'(p) \neq 0$ prove that there exist a $\delta \succ 0$ such that the sequence $\{p_k\}$ defined by iteration

$$p_k = p_{k-1} - \frac{f(p_{k-1})}{f'(p_{k-1})}$$ for k = 1, 2... converges to p for any initial approximation $p_0 \in [p - \delta, p + \delta]$. **6**

B) Let f (x) = $(x – 2)^4$

i) Find Newton-Raphson formula.

ii) Start with p_0 = 2.1 and compute compute p_1, p_2, p_3.

iii) Is the sequence converging quadratically or linearly ? **5**

C) Solve the system of equation **5**

$$\begin{bmatrix} 2 & 1 & 1 & -2 \\ 4 & 0 & 2 & 1 \\ 3 & 2 & 2 & 0 \\ 1 & 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -10 \\ 8 \\ 7 \\ -5 \end{bmatrix}$$

Using the Gauss elimination method with partial pivoting.

3. A) Explain Gaussian elimination method for solving a system of m equation in n knows. **6**

B) Find the Jacobin J (X, Y, Z) of order 3 3 at the point (1, 3, 2) for the functions

f_1 (X, Y, Z) = $X^3 – Y^2 + Y – Z^4$, f_2 (X, Y, Z) = XY + YZ + XZ. f_3 (X, Y, Z) = $\frac{Y}{XZ}$. **5**

C) Compute the divided difference table for f (x) = 3×2^x

x :	–1.0	0.0	1.0	2.0	3.0
f(x) :	1.5	3.0	6.0	12.0	24.0

Write down the Newton's polynomial P_4 (x). **5**

4. A) Assume that $f \in C^{N+1}[a,b]$ and x_0, x_1 ....$x_N \in [a, b]$ are N + 1 nodes. If $x \in [a, b]$ then prove that f (x) = P_N (x) + E_N (x).

where P_N (x) is a polynomial that can be used to approximate f (x) and E_N (x) is the corresponding error in the approximation. **6**

B) Consider the system :

$5x - y + z = 10$

$2x + 8y - z = 11$

$-x + y + 4z = 3$ $\quad\quad P_0 = 0$

And use Gauss-Seidel iteration to find P_1, P_2, P_3. Will this iteration convergence to the solution ? **5**

C) Find the triangular factorization A = LU for the matrix **5**

$$A = \begin{bmatrix} 1 & 1 & 0 & 4 \\ 2 & -1 & 5 & 0 \\ 5 & 2 & 1 & 2 \\ -3 & 0 & 2 & 0 \end{bmatrix}$$

5. A) Assume that $f \in C^5[a,b]$ and that $x-2h, x-x, x, x+h, x+2h \in [a,b]$ prove that

$$f'(x) \bullet \frac{-f(x)+2h+8f(x+h)-8f(x-h)+f(x-2h)}{12h}$$ **6**

B) Let f (x) = x^3 find approximation for $f'(2)$. Use formula in Part (a) with h = 0.05. **5**

C) Use Newton's method with the starting value $(p_0, q_0) = (2.00,.25)$ compute (p_1, q_1), (p_2, q_2) for the nonlinear system : **5**

$x^2 - 2x - y + 0.5 = 0$, $x^2 + 4y^2 - 4 = 0$.

6. A) Assume that $X_j = X_0 + h_j$ are equally spaced nodes and $f_j = f(x_j)$. Derive the Quadrature formula $\int_{x_0}^{x_2} f(x) \bullet \frac{h}{3}(f_0 + 4f_1 + f_2)$. **6**

B) Let f (x) = $\frac{8x}{2^x}$

Use cubic Langrange's interpolation based on nodes

$x_0 = 0$, $x_1 = 1$ $x_2 = 2$, $x_3 = 3$ to approximate f (1.5). **5**

C) Consider f (x) = 2 + sin $(2\sqrt{x})$. Investigate the error when the composite trapezoidal rule is used over [1, 6] and the number of subinterval is 10. **5**

7. A) Use Euler's method to solve the I V P

   $y' = -ty$ over [0, 0.2] with y (0) = 1. Compute y_1, y_2 with h = 0.1

   Compare the exact solution y (0.2) with approximation. **8**

   B) Use the Runge-kutta method of order N = 4 to solve the I.V.P. $y' = t^2 - y$ over [0, 0.2] with y (0) = 1, (taken h = 0.1)

   Compare with $y(t) = -e^{-t} + t^2 - 2t + 2$. **8**

8. A) Use power method to find the dominant Eigen value and Eigen vector for the

   Matrix $A = \begin{bmatrix} 0 & 11 & -5 \\ -2 & 17 & -7 \\ -4 & 26 & -10 \end{bmatrix}$ **8**

   B) Use Householder's method to reduce the following symmetric matrix to trigonal form

   $$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$ **8**

Seat No.	

M.A./M.Sc. (Semester – IV) Examination, 2012
MATHEMATICS
MT-801 : Field Theory (2008 Pattern)

Time : 3 Hours Max. Marks : 80

N.B. : *1) Attempt* ***any five*** *questions.*
2) Figures to the ***right*** *indicate* ***full*** *marks.*

1. a) Let $f(x) \in \mathbb{Z}[x]$ be primitive, then prove that f(x) is reducible over $\mathbb{Q}$ if and only if it is reducible over $\mathbb{Z}$. **5**

   b) Show that $x^3 - x - 1 \in \mathbb{Q}[x]$ is irreducible over $\mathbb{Q}$. **5**

   c) Find all irreducible polynomials of degree 3 over $\mathbb{Z}_2$. **6**

2. a) Let E be a finite extension of a field F and K be a finite extension of E, then prove that K is also finite extension of F. **6**

   b) Prove that finite extension E of a finite field F is a simple extension. **5**

   c) Establish the equality $\mathbb{Q}\left(\sqrt{2}, \sqrt{5}\right) = \mathbb{Q}\left(\sqrt{2} + 3\sqrt{5}\right)$. **5**

3. a) If a multiplicative group F* of non-zero elements of a field F is cyclic, then prove that F is finite. **6**

   b) Give an example of a polynomial $f(x) \in F[x]$ of degree n such that the splitting field E of f(x) over F has degree n. **5**

   c) Find the splitting field of $f(x) = x^4 - z \in \mathbb{Q}[x]$. **5**

4. a) Let F be a field, and let $\sigma : F \to L$ be an embedding of F into an algebraically closed field L. Let $E = F(\alpha)$ be an algebraic extension of F, then prove that σ can be extended to an embedding $\eta : E \to L$. How many such extensions are possible ? Explain. **6**

   b) Prove that every finite extension of a finite field is normal. **5**

   c) Let F be a finite field with 625 elements. Does there exist a subfield of F with 125 elements ? With 25 elements ? Justify. **5**

5. a) Let H be a finite subgroup of a group of automorphisms of a field E, then prove that $[E:E_H] = |H|$, $|H|$ denotes the order of H. **6**

 b) If F is a finite field of characteristic P, then show that each element $a \in F$ has a unique p^{th} root $\sqrt[p]{a} \in F$. **5**

 c) Show that the group of $\mathbb{Q}$-auto morphisms of $\mathbb{Q}\left(\sqrt[3]{2}\right)$ is a trivial group. **5**

6. a) Let E be a finite separable extension of a field F and E is normal extension of F, then prove that F is the fixed field of G(E/F). **8**

 b) Let $E = \mathbb{Q}\left(\sqrt[3]{2}, \omega\right)$, where $w^3 = 1$, $w \neq 1$. Let σ_0 be the identity automorphism of E and let σ_1 be an automorphism of E such that $\sigma_1(w) = w^2$ and $\sigma_1\left(\sqrt[3]{2}\right) = W\left(\sqrt[3]{2}\right)$. If $G = \{\sigma_0, \sigma_1\}$, the show that $E_G = \mathbb{Q}\left(\sqrt[3]{2}\, w^2\right)$. **8**

7. a) State only the fundamental theorem of Galois theory. **4**

 b) Find the galois group $G(K/\mathbb{Q})$, where $K = \mathbb{Q}\left(\sqrt{3}, \sqrt{5}\right)$. **6**

 c) Is $\mathbb{R}[x] \big/ < x^2 - 2 >$ a field ? Justify your answer. **3**

 d) Find the basis of $\mathbb{Q}\left(\sqrt[4]{2}\right)$ over $\mathbb{Q}$. **3**

8. a) Prove that a real number a is constructible from $\mathbb{Q}$ if and only if (a, 0) is constructible point from $\mathbb{Q} \times \mathbb{Q}$. **5**

 b) Show that if an irreducible polynomial p(x) in F[x] over a field F has a root in radical extension of F, then p(x) is solvable by radicals over F. **6**

 c) Find the basis of $\mathbb{Q}\left(\sqrt[3]{2}, \sqrt{5}\right)$ over $\mathbb{Q}$. **5**

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Seat No.	

M.A./M.Sc. (Semester – IV) Examination, 2012
MATHEMATICS
MT-802 : Combinatorics (New)
(2008 Pattern)

Time : 3 Hours   Max. Marks : 80

N.B. : *1) Attempt **any five** questions.*
*2) Figures to the **right** indicate **full** marks.*

1. A) What is the probability of randomly choosing a permutation of the 10 digits 0, 1, 2, ...9 in which : **6**

   a) An odd digit is in the first position and 1, 2, 3, 4 or 5 is in the last position.

   b) 5 is not in the first position and 9 is not in the last position.

   B) Prove by combinatorial argument that **6**

   $$\binom{r}{r}+\binom{r+1}{r}+\binom{r+2}{r}+\ldots\ldots+\binom{n}{r}=\binom{n+1}{r+1}$$

   Hence, evaluate the sum

   $1\times2\times3 + 2\times3\times4 + ...+ (n-2)(n-1)n$.

   C) Find the rook polynomial for a full $n\times n$ board. **4**

2. A) How many 8- digit sequences are there involving exactly six different digits ? **6**

   B) Find ordinary generating function whose coefficient a_r equals r. Hence evaluate the sum 0+1+2 + ...+ n. **6**

   C) How many nonnegative integer solutions are there to the inequalities $x_1 + x_2 + ... + x_6 \le 20$ and $x_1 + x_2 + x_3 \le 7$? **4**

P.T.O.

3. A) Use generating functions to find number of ways to split 6 copies of one book, 7 copies of a second book and 11 copies of a third book between two teachers if each teacher gets 12 books and each teacher gets at least 2 copies of each book. **6**

B) How many permutations of the 26 letters are there that contain none of the sequences MATH, RUNS, FROM, or JOE ? **6**

C) Find a generating function for the number of integers between 0 and 9,99,999 whose sum of digits is r. **4**

4. A) How many ways are there to make an r- arrangement of pennies, nickels, dimes and quarters with at least one penny and an odd number of quarters ? **6**

B) Solve the recurrence relation **6**

$a_n = 3a_{n-1} - 3a_{n-2} + a_{n-3}$, $a_0 = a_1 = 1$, $a_2 = 2$.

C) Show that any subset of eight distinct integers between 1 and 14 contains a pair of integers k, l such that k divides l. **4**

5. A) How many ways are there to assign 20 different people to three different rooms with at least one person in each room ? **6**

B) Find a rook polynomial to send 4 different birthday cards denoted by c_1, c_2, c_3, c_4 to four persons p_1, p_2, p_3, p_4 if p_1 would not like cards c_2 or c_3 ; p_2 would not like cards c_1 or c_4; p_3 would not like cards c_2 or c_4 ; p_4 would not like card c_3. **6**

C) How many sequences of length 5 can be formed using the digits 0, 1, 2, ...9 with the property that exactly two of the 10 digits appear. (e.g. 05550). **4**

6. A) Suppose a bookcase has 200 books 70 in French, and 100 about mathematics. How many non-French books not about mathematics are there if **6**

i) There are 30 French mathematics books ?

ii) There are 60 French nonmathematics books ?

B) Solve the recurrence relation **6**

$a_n = -na_{n-1} + n!$ given $a_0 = 1$.

C) Show that any subset of n + 1 distinct integers between 2 and 2n ($n \geq 2$) always contains a pair of integers with no common divisor. **4**

7. A) Using generating functions, solve the recurrence relation. **6**
$a_n = a_{n-1} + n(n-1)$; $a_0 = 1$.

B) How many 10- letter words are there in which each of the letters e, n, r, s occur **6**

i) at most once ?

ii) at least once ?

C) How many ways are there to distribute eight distinct balls into six boxes with the first two boxes collectively having at most four balls. **4**

8. A) Five officials $O_1, O_2, \ldots, O_5$ are to be assigned five different city cars an Escort, a Lexus, a Nissan, a Taurus and a Volvo. **8**

O_1 will not drive an Escort or Volvo;

O_2 will not drive Lexus or Nissan; O_3 will not drive Nissan; O_4 will not drive Escort or volvo; O_5 will not drive Nissan. How many ways are there to assign the officials to different cars ?

B) Find and solve a recurrence relation for the number of ways to arrange flags on an n-foot flagpole using three types of flags: red flags 2 feet high, yellow flags 1 foot high and blue flags 1 foot high. **8**

M.A./M.Sc. (Semester – IV) Examination, 2012
MATHEMATICS
MT-802 : Hydrodynamics (Old)

Time : 3 Hours Max. Marks : 80

N.B. : *1) Attempt* ***any five*** *questions.*
2) ***Figures*** *to the* ***right*** *indicate* ***full*** *marks.*

1. a) What are stream lines ? Are stream lines and paths of particles of a fluid always the same ? Give reason. **6**

b) A three dimensional velocity field is given by

$$u = xy^2t,\ v = \frac{1}{3}y^3t^3,\ w = \frac{1}{2}xyz^2t^2.$$

Determine the total acceleration at (1, 1, 1) at t = 1 sec. **6**

c) Write a note on physical interpretation of stream function. **4**

2. a) Prove that $\int \frac{dp}{\rho} + \frac{1}{2}q^2 + \Omega = c$ when the motion is steady and the velocity potential does not exist, Ω being the potential function from which the external forces are derivable. **8**

b) Show that the variable ellipsoid $\frac{x^2}{a^2k^2t^4} + kt^2\left[\left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2\right] = 1$

is a possible form for the boundary surface of a liquid at any time t. **8**

3. a) Find the radial and transverse components of velocity of a fluid particle in terms of velocity potential and stream function. **6**

b) A flow pattern is obtained by the superposition of two flow patterns 1 and 2 defined by stream function

$\psi_1 = \frac{y^3}{3} - x^2y + 2xy$ and velocity potential $\phi_2 = 2x^2 - 2y^2$

Show that both of the flow patterns are irrotational and obtain the velocity components for the combined flow. **10**

4. a) State and prove Circle theorem. **6**

b) A circular cylinder is fixed across a stream of velocity V with circulation K round the cylinder. Show that the maximum velocity in the liquid is $2V + \frac{k}{2\pi a}$, where a is the radius of cylinder. **10**

5. a) State and prove theorem of Kutta and Joukowski. **8**

b) Two pairs of vortices each of strength K are situated at ($\pm$ a, 0) and a point vortex of strength $\frac{-K}{2}$ is situated at origin. Show that the liquid motion is stationary. Determine stagnation point. **8**

6. a) Define : Vortex lines, Vortex tube and Vortex filament. Determine stream lines in case of Vortex pair. **8**

b) Between the two fixed boundaries $\theta = \frac{\pi}{6}$ and $\theta = \frac{-\pi}{6}$, there is a two dimensional liquid motion due to a source at a point r = c, $\theta = \alpha$ and a sin K at origin, absorbing water at the same rate as the source produces it. Find the stream function and show that one of the stream lines is a part of the curve $r^3 \sin 3\alpha = c^3 \sin 3\theta \cdot$ **8**

7. a) Obtain the relation between stress and rate of strain components. **8**

b) Show that stress tensor is symmetric. **8**

8. Write explanatory notes on **any two** : **16**

a) Blasius theorem.

b) Lagrangian and Eulerian methods.

c) Image of a source in a circle.

d) Karman's vortex sheet.

Seat No.	

M.A./M.Sc. (Semester – IV) Examination, 2012
MATHEMATICS
MT 803 : Differential Manifolds
(2008 Pattern)

Time : 3 Hours   Max. Marks : 80

N.B. : *1) Attempt* ***any five*** *questions.*
2) ***All*** *questions carry* ***equal*** *marks.*

1. a) Let W be a K-dimensional subspace of $\mathbb{R}^n$. Show that there exists an orthogonal transformation on $\mathbb{R}^n$ that carries W onto $\mathbb{R}^k \times 0$. **8**

b) Let $X = \begin{pmatrix} 1 & 0 & 1 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{pmatrix}$. Find V(X). **4**

c) Give an example of a 1 manifold in $\mathbb{R}^3$. **4**

2. a) Let M be a manifold in $\mathbb{R}^n$, and let $\alpha : U \to V$ be a coordinate patch on M. If U_0 is a subset of U that is open in U, then show that the restriction of α to U_0 is also a coordinate patch on M. **6**

b) Show that the function $\alpha :[0, 1] \to S^1$ given by α (t) = (cos2πt, Sin2πt) is not a coordinate patch on S^1. **6**

c) Give an example of a compact 2-manifold without boundary. **4**

3. a) If the support of f can be covered by a single coordinate patch, then show that the integral $\int_M f \, dv$ is well defined, independent of the choice of coordinate patch. **6**

b) Find area of the 2-sphere $S^2(a)$. **6**

c) Give an example of a 2-tensor on $\mathbb{R}^4$. **4**

P.T.O.

4. a) If f is an alternating K-tensor and g is an alternating l-tensor, then show that $g \wedge f = (-1)^{kl} \ f \wedge g$. **8**

   b) Show that $f(x, y) = x_i y_j - x_j y_i$ is an alternating tensor on $\mathbb{R}^n$. **4**

   c) Find basis and dimension of the space A^k (v) of alternating K-tensors on V; where dimension of V is n. **4**

5. a) Let M be a K-manifold in $\mathbb{R}^n$ and $P \in M$. Define the tangent space to M at p and show that it is independent of the choice of the coordianate patch at p. **6**

   b) Consider the form :

   W = xydx + 3 dy – yz dz. Verify by direct computation that d(dw) = 0. **6**

   c) Define the terms :

   i) Exact form ii) Closed form. **4**

6. a) If w and η are forms of orders k and l respectively, then prove that $d(w \wedge \eta) = dw \wedge \eta + (-1)^k w \wedge d\eta$. **8**

   b) In $\mathbb{R}^3$, let w = xzdx + 2x dy – xdz. Let $\alpha : \mathbb{R}^2 \to \mathbb{R}^3$ be given by the equation $\alpha(u, v) = (u^2, u + v, uv)$.

   Calculate :

   i) dw, ii) $\alpha^* w$ iii) $\alpha^*(dw)$ iv) d ($\alpha^* w$) directly. **8**

7. a) Define orientable manifold. Prove that if M is an orientable k-manifold (k>1) with non-empty boundary, then ∂m is orientable. **8**

   b) Let $A = (0, 1)^2$. Let $\alpha : A \to \mathbb{R}^3$ be given by the equation $\alpha(u, v) = (u, v, u+v)$.

   Let y be the image set of α. Evaluate $\int_{Y\alpha} x_2 \, dx_2 \wedge dx_3 + x_1 x_3 \, dx_1 \wedge dx_3$. **8**

8. a) State Stokes' theorem and deduce Green's theorem from it. **8**

   b) If M is an orientable (n–1) manifold in $\mathbb{R}^n$, then define unit normal field to M w.r.t. given orientation. **4**

   c) Give an example of a non-orientable manifold. **4**

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Seat No.	

M.A./M.Sc. (Semester – IV) Examination, 2012
MATHEMATICS
MT – 804 : Algebraic Topology (New)
(2008 Pattern)

Time : 3 Hours Max. Marks : 80

N.B. : *1) Attempt* ***any five*** *questions.*
2) Figures to the ***right*** *indicate* ***full*** *marks.*

1. a) Let $i : S^{n-1} \to B^n$ be the inclusion map. Show that $f : B^n \to S^{n-1}$ with $f \circ i = I$ if and only if the identity map $I : S^{n-1} \to S^{n-1}$ is homotopic to a constant map. **6**

   b) Prove that the relation of being homotopic relative to a set A is an equivalence relation. **5**

   c) Let f, g : $X \to S^n$ be continuous mappings such that $f(x) \neq -g(x)$ for all $x \in X$. Show that f is homotopic to g. **5**

2. a) Prove that if Y is contractible, then every continuous mapping $f : X \to Y$ is homotopic to a constant map. **6**

   b) Define a strong deformation retract A of a topological space X. Prove that S^n is a strong deformation retract of $\mathbb{R}^{n+1} - \{0\}$. **5**

   c) Show that a retract of a Housdorff space is a closed subset. **5**

3. a) Prove that if f is any path, then $f * \bar{f}$ and $\bar{f} * f$ are homotopic to null paths. **6**

   b) Let $f : [0, 1] \to X$ be a path in X and let $g : [0, 1] \to [0, 1]$ be a continuous map. Show that f is homptopic to $f \circ g$ relative to {0, 1}. **5**

   c) Show that every path connected space is connected. Is converse true ? Justify your answer. **5**

P.T.O.

4. a) Let $x_0, x_1 \in X$. Suppose there is a path in X from x_0 to x_1. Prove that the fundamental groups $\pi_1(X, x_0)$ and $\pi_1(X, x_1)$ are isomorphic. **6**

b) Prove that the fundamental group of the real projective plane is a cyclic of order two. **5**

c) Let X and Y be of same homotopy type and $\phi : X \to Y$ be a homotopy equivalence. Prove that $\phi^* : \pi_1(X, x) \to \phi_1(Y, \phi(x))$ is an isomorphism for any $x \in X$. **5**

5. a) Show that the fundamental group of the circle S^1 is the additive group of integers. **8**

b) Find the fundamental groups of the three spaces : $\mathbf{R}^n$, $\mathbf{R}^2 - \{0, 0\}$, and $S^1 \times \mathbf{R}$. **8**

6. a) Define a covering map. Show that a covering map is a local homeomorphism. **6**

b) Give an example of a nonidentity covering map from S^1 onto S^1. **5**

c) Let $p : \tilde{X} \to X$ and $q : \tilde{Y} \to Y$ be covering maps. Show that $p \times q : \tilde{X} \times \tilde{Y} \to X \times Y$ is a covering map. **5**

7. a) Let $p : \tilde{X} \to X$ be a fibration with unique path lifting. Suppose that f and g are paths in $\tilde{X}$ with f (0) = g (0) and pf ~ pg. Prove that f ~ g. **6**

b) Let $p : E \to B$ be a fibration. Prove that p (E) is a union of path components of B. **5**

c) A fibration has unique path lifting if every fiber has non-null path. **5**

8. a) Prove that the closed ball B^n $(n \geq 1)$ has the fixed point property. **8**

b) Prove that every complex has a barycentric subdivision. **8**

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Seat No.	

M.A./M.Sc. (Semester – IV) Examination, 2012
MATHEMATICS
MT – 804 : Mathematical Methods – II (Old)

Time : 3 Hours Max. Marks : 80

N.B. : *i) Attempt* ***any five*** *questions.*
ii) Figures to the ***right*** *indicate* ***full*** *marks.*

1. a) Define :
   i) Fredholm integral equation of the first kind.
   ii) Symmetric Kernels. **4**

   b) Show that the function $u(x) = (1+x^2)^{\frac{-3}{2}}$ is a solution of the integral equation

   $u(x) = \frac{1}{1+x^2} - \int_0^x \frac{t}{1+x^2} u(t)\, dt$. **6**

   c) Explain the method to find the solution of the integral equation

   $\phi(s) = \lambda \int_a^b K(s,t)\, \phi(t)\, dt$, where K (s, t) is separable Kernel. **6**

2. a) Convert the following initial value problem into volterra integral equation

   $\frac{d^2y}{dx^2} + xy = 1$, y (0) = y' (0) = 0. **8**

   b) Reduce the following boundary value problem into an integral equation

   $\frac{d^2y}{dx^2} + xy = 1$ with y (0) = y (1) = 0. **8**

3. a) Find eigen values and eigen vectors or eigen functions of the homogeneous Fredholm integral equation of the second kind $\phi(x) = \lambda \int_0^1 (2xt - 4x^2)\, \phi(t)\, dt$. **8**

   b) Find the iterated Kernels for the Kernel K (x, t) = $e^x \cos t$; a = 0, b = π. **8**

4\. a) Solve $u(x) = e^x - \frac{1}{2}e + \frac{1}{2} + \frac{1}{2}\int_0^1 u(t)\,dt$ by resolvent Kernel. **8**

b) Find the Neumann series for the solution of the integral equation

$y(x) = 1 + x + \lambda \int_0^x (x - t)\,y(t)\,dt$. **8**

5\. a) State and prove isoperimetric problem. **8**

b) Find the extremal of the functional $\int_{x_0}^{x_1} \left(y'^2 / x^3 \right) dx$. **8**

6\. a) Prove that $\frac{d}{dx}\frac{\partial f}{\partial y'} - \frac{\partial f}{\partial y} = 0$ (Euler-Lagrange's equation) with usual notations. **8**

b) Let $\psi_1(s), \psi_2(s), \ldots$ be a sequence of functions whose norms are all below a fixed bound M and for which the relation $\psi_n(s) - \lambda \int K(s, t)\,\psi_n(t)\,dt = 0$ holds in the sense of uniform convergence. Prove that the functions $\psi_n(s)$ form a smooth sequence of functions with finite asymptotic dimension. **8**

7\. a) Solve the symmetric integral equation $y(x) = x^2 + 1 + \frac{3}{2}\int_{-1}^{1} \left(xt + x^2 t^2\right) y(t)\,dt$ by using Hilbert Schmidt theorem. **10**

b) State and prove Harr theorem. **6**

8\. a) State and prove principal of Least action. **8**

b) Find the path on which a particle in the absence of friction, will slide from one point to another in the shortest time under the action of gravity. **8**

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Seat No.	

M.A./M.Sc. (Semester – IV) Examination, 2012
MATHEMATICS
MT-805 : Lattice Theory
(2008 Pattern)

Time : 3 Hours   Max. Marks : 80

N.B. *: 1) Attempt* ***any five*** *questions.*
2) Figures to the ***right*** *indicate* ***full*** *marks.*

1. a) Let A be the set of all real valued functions defined on a set X; for f, g $\in$ A, set f $\leq$ g to mean f(x) $\leq$ g(x) for all x $\in$ X. Prove that $\langle A; \leq \rangle$ is a lattice. **5**

   b) Define a complete lattice and prove that a poset $\langle L; \leq \rangle$ is a complete lattice if and only if inf H exists for any subset H of L. **5**

   c) Define a congruence relation on a lattice L and find all congruence relations of a non-modular lattice N_5. **6**

2. a) Prove that if a lattice L is distributive, then Id(L), the ideal lattice of L, is distributive. **5**

   b) Prove that a lattice L is distributive if and only if for any two ideals I, J of L, $I \vee J = \{ i \vee j \mid i \in I, j \in J \}$. **6**

   c) Show that an ideal P is a prime ideal of a lattice L if and only if L\P is a dual ideal. **5**

3. a) Let L be a pseudocomplemented lattice. Assuming $S(L) = \{ a^* \mid a \in L \}$ is a lattice, prove that S(L) is distributive. **4**

   b) Prove that every complete lattice is bounded. Is the converse true ? Justify your answer. **4**

   c) Prove that a lattice is modular if and only if it does not contain a pentagon (N_5) as a sublattice. **8**

P.T.O.

4. a) Prove that in a finite lattice L, every element is the join of join-irreducible elements. **5**

b) Show that the following inequalities hold in any lattice. **5**

1) $(x \wedge y) \vee (x \wedge z) \leq x \wedge (y \vee z)$; 2) $(x \wedge y) \vee (x \wedge z) \leq x \wedge (y \vee (x \wedge z))$.

c) Prove that every maximal ideal of a distributive lattice is prime but the converse need not hold. **6**

5. a) Prove that every maximal Chain C of the finite distributive lattice L is of length $|J(L)|$ where J(L)is the set of all join-irreducible elements of L. **5**

b) Prove that every isomorphism is an isotone map. Is the converse true ? Justify. **4**

c) State and prove Nachbin Theorem. **7**

6. a) Let L be a finite distributive lattice. Then show that the map $\phi : a \to r(a)$ is an isomorphism between L and H(J(L)), the hereditary subsets of the set of join-irreducibles of L. **8**

b) Let L be a distributive lattice, let I be an ideal, let D be a dual ideal of L, and let $I \cap D = \phi$. Then prove that there exists a prime ideal P of L such that $P \supseteq I$ and $P \cap D = \phi$. **8**

7. a) State and prove Jordan-Hölder Theorem for semimodular lattices. **8**

b) Let L be a complete lattice and $f : L \to L$ be an isotone map. Then prove that there exists $a \in L$ such that f(a) = a. **5**

c) Prove that the ideal lattice of a Boolean lattice need not be Boolean. **3**

8. a) Prove that any finite distributive lattice is pseudocomplemented. **4**

b) Prove that the absorption identities imply the idempotency of $\wedge$ and $\vee$. **4**

c) Let L be a finite distributive lattice and S(L) = {a*|a$\in$L}. Then prove that **6**

i) $a \in S(L)$ if and only if a = a** ii) a, b $\in$ S(L) implies $a \wedge b \in S(L)$.

d) Find a bounded distributive lattice L such that S(L) = {0, 1} and |L| > 3. **2**
